Reconciling Parker Solar Probe observations and magnetohydrodynamic theory: *à la* Kolmogorov vs. *à la* Iroshnikov-Kraichnan scale-invariance

Tommaso Alberti[a,1], **Simone Benella**[a], **Giuseppe Consolini**[a], **Mirko Stumpo**[b,a], **and Roberto Benzi**[b]

[a]INAF-Istituto di Astrofisica e Planetologia Spaziali, 00133 Roma, Italy; [b]Dip. Fisica, Università degli Studi di Roma Tor Vergata, 00133 Roma, Italy

The Parker Solar Probe mission provides a unique opportunity to characterize several features of the solar wind at different heliocentric distances. Recent findings have shown the existence of a different scale-invariant nature when moving away from the Sun. Here we provide, for the first time, how to reconcile these observational results on the nature of the radial evolution of the magnetic and velocity field fluctuations across the inertial range with two scenarios drawn from the magnetohydrodynamic theory. In details, we evidence (i) a magnetically–dominated scenario up to 0.4 AU and (ii) a fluid–like at larger distances. The observed breakdown is the result of the radial evolution of magnetic field fluctuations and plasma thermal expansion affecting the distribution of between magnetic and kinetic fluctuations. The two scenarios can be reconciled with those of Iroshnikov-Kraichnan and Kolmogorov pictures of turbulence in terms of an evolving nature of the coupling between fields. Our findings have important implications for turbulence studies and modeling approaches.

Solar wind | Magnetohydrodynamic turbulence | Parker Solar Probe

Since 2018 the Parker Solar Probe (PSP) mission is collecting solar wind plasma and magnetic field data through the inner heliosphere, reaching the closest distance to the Sun ever reached by any previous mission (1). Thanks to the PSP journey around the Sun (it has completed 8 orbits) a different picture has been drawn for the near-Sun solar wind with respect to the near-Earth one (2–5). Different near-Sun phenomena have been frequently encountered, as the emergence of magnetic field flips, i.e., the so-called switchbacks (6), kinetic-scale current sheets (7), and a scale-invariant population of current sheets between ion and electron inertial scales (8). Going away from the Sun, it has been shown a radial evolution of the scaling properties of solar wind turbulence in the heliocentric range of distances 0.17–0.8 astronomical units (AU) (9). Although the near-Sun solar wind shares different properties with the near-Earth one (10, 11), significant differences have been found in the variance of magnetic fluctuations (about two orders of magnitude), in the compressive component of inertial range turbulence, and in the imbalance between inward and outward Alfvénic fluctuations. (9) also reported a steepening of the power spectral density spectra, moving from –3/2 close to the Sun to –5/3 at distances larger than 0.4 AU. In a similar way, (12) firstly reported a breakdown of the scaling properties of the energy transfer rate, likely related to the breaking of the phase-coherence of inertial range fluctuations. These findings, mainly related to the physics of the inertial range usually described in the framework of magnetohydrodynamics (MHD), have been interpreted as an increase in the efficiency of the nonlinear energy cascade mechanism when moving away from the Sun. Indeed, recent findings by (13) reported a concurrent effect between large field gradients and small fluctuations close to the Sun, necessary to dissipate the excess of kinetic energy across the inertial range, while small fluctuations with a regular topology are mainly observed at BepiColombo orbit near 0.6 AU. All these features shed new light into the radial evolution of scaling properties that urge to be considered in expanding models of the solar wind (14, 15), also to reproduce and investigate the role of proton heating and anisotropy of magnetic field fluctuations (16). Furthermore, a novel framework needs to be provided to explain the transition between singular and regular topological structures in terms of cascade models (17–19) and (multi)fractal approaches (20–23).

In this work we provide a theoretical and observational framework to explain the observed radial features by Parker Solar Probe via the Elsässer fields radial evolution. We find evidence of two different scenarios: a magnetically–dominated up to 0.4 AU and a fluid–like at larger distances. The observed breakdown is the result of the radial evolution of the distribution between magnetic and kinetic fluctuations. The two scenarios can be reconciled with those of Iroshnikov-Kraichnan and Kolmogorov pictures of turbulence in terms of the radial evolution of the coupling between fields.

Theoretical background

We start our theoretical part by writing the incompressible MHD equations

$$\partial_t \mathbf{z}^{\pm} + \left(\mathbf{C_A} \cdot \nabla\right) \mathbf{z}^{\pm} + \left(\mathbf{z}^{\mp} \cdot \nabla\right) \mathbf{z}^{\pm} = -\nabla p + \nu^{\pm} \nabla^2 \mathbf{z}^{\pm}, \qquad [1]$$

where $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$ are the Elsasser variables (24), being $\mathbf{v}$ the velocity field and $\mathbf{b} = \frac{\mathbf{B}}{\sqrt{\mu_0 \rho_0}}$ the magnetic field in Alfvén units, $C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$ is the background Alfvén speed, p is the kinetic pressure, and $\nu^{\pm}$ are dissipative coefficients. The Elsässer variables describe the inward- and outward-propagating Alfvénic fluctuations (24). Since previous findings on the inertial range are mainly linked with the nonlinear term and its radial evolution we focus our attention to the term $\left(\mathbf{z}^{\mp} \cdot \nabla\right) \mathbf{z}^{\pm}$. One of the most striking features observed by PSP when approaching the Sun is the increase of the ratio between outward and inward fluctuations from $|\mathbf{z}^{+}|/|\mathbf{z}^{-}| \sim 1$ to $|\mathbf{z}^{+}|/|\mathbf{z}^{-}| \sim 15$, although $\mathbf{z}^{\pm}$ show a similar spectral exponent (9). This means that close to the Sun we are in an unbalanced scenario in which $|\mathbf{z}^{+}| \gg |\mathbf{z}^{-}|$, evolving towards a symmetric state $|\mathbf{z}^{+}| \sim |\mathbf{z}^{-}|$ far away. These two states, i.e., $|\mathbf{z}^+| \gg |\mathbf{z}^-|$ close to the Sun and $|\mathbf{z}^+| \sim |\mathbf{z}^-|$ at large distances, can be related to a different nature of the coupling between $\mathbf{v}$ and $\mathbf{b}$. Indeed, the condition $|\mathbf{z}^+| \sim |\mathbf{z}^-|$ means

$$|\mathbf{v}+\mathbf{b}| \sim |\mathbf{v}-\mathbf{b}| \tag{2}$$

traducing into

$$|\mathbf{v}|^2 + |\mathbf{b}|^2 + 2\mathbf{v}\cdot\mathbf{b} \sim |\mathbf{v}|^2 + |\mathbf{b}|^2 - 2\mathbf{v}\cdot\mathbf{b} \tag{3}$$

giving rise to the condition $\mathbf{v}\cdot\mathbf{b} = 0$, i.e., $\mathbf{v} \perp \mathbf{b}$. On the other hand the condition $|\mathbf{z}^+| \gg |\mathbf{z}^-|$ traduces into

$$|\mathbf{v}|^2 + |\mathbf{b}|^2 + 2\mathbf{v}\cdot\mathbf{b} \gg |\mathbf{v}|^2 + |\mathbf{b}|^2 - 2\mathbf{v}\cdot\mathbf{b} \tag{4}$$

giving rise to the condition $\mathbf{v}\cdot\mathbf{b} = 1$, i.e., $\mathbf{v} \parallel \mathbf{b}$. These findings suggest to revise an old view by *Dobrowolny et al.* (25) according to which an initially asymmetric MHD turbulence $|\mathbf{z}^+| \gg |\mathbf{z}^-|$, in absence of nonlinear interactions, relaxes toward a state characterized by the absence of one of the possible modes $\mathbf{z}^+$ or $\mathbf{z}^-$. PSP observations suggest that the relaxation is from an initially asymmetric state toward a symmetric one that can be linked to a different nature of the $\mathbf{v}$–$\mathbf{b}$ coupling. The two states can be also explained in terms of measurable quantities as the normalized cross-helicity σ_C and the normalized residual energy σ_R

$$\sigma_C = \frac{2\langle \mathbf{v}\cdot\mathbf{b}\rangle}{\langle \mathbf{v}^2\rangle + \langle \mathbf{b}^2\rangle}, \tag{5}$$

$$\sigma_R = \frac{\langle \mathbf{v}^2\rangle - \langle \mathbf{b}^2\rangle}{\langle \mathbf{v}^2\rangle + \langle \mathbf{b}^2\rangle}. \tag{6}$$

σ_C is a measure of the energy balance between outward and inward Alfvénic fluctuations, while σ_R measures the balance between kinetic and magnetic energy. $\sigma_C = \pm 1$ evidences the presence of only one component (+: outward, −:inward), $|\sigma_C| < 1$ corresponds to the presence of both components and/or to non-Alfvénic fluctuations, while $\sigma_R = \pm 1$ evidences the existence of magnetic–/kinetic–only fluctuations, with $\sigma_R = 0$ meaning equipartition. Based on PSP observations and on the two scenarios that can be drawn (i.e., $|\mathbf{z}^+| \gg |\mathbf{z}^-|$ close to the Sun and $|\mathbf{z}^+| \sim |\mathbf{z}^-|$ at large distances), moving away from the Sun we expect to transit from a state $(\sigma_C, \sigma_R) = (\rightarrow 1, \lesssim 0)$ toward a state $(\sigma_C, \sigma_R) = (\sim 0, \rightarrow 1)$. In the following we explore our conjectures by evaluating the joint probability of occurrence between pairs of values (σ_C, σ_R) at different heliocentric distances.

Parker Solar Probe observations

We use PSP magnetic field and plasma measurements (2, 26) in the time interval from December 2020 to October 2021, i.e., during the three PSP perihelia of 2021. The choice of this time interval is based on a good quality of data and a good coverage at different heliocentric distances. All data are at 1-minute time resolution, forming a dataset of $N = 437760$ data points, covering the heliocentric range of distances between ∼0.1 and 0.8 AU.

Figure 1 reports the plasma bulk speed V, the Alfvén speed V_A, and the PSP radial distance to the Sun R, respectively. A dependence on the heliocentric distance R of the Alfvén speed seems to be present, while the solar wind speed is independent on R. This suggests that the velocity field has reached its fully-developed state, while the Alfvén field radially evolves according to the large-scale configuration of the Parker spiral and to the expansion of the solar wind plasma through the innermost Heliosphere as an outward-streaming gas (27). These results are confirmed by looking at Fig. 2 where the radial dependence of the ratio between the Alfvén and the plasma speeds as well as their variances are reported.

A scaling-law behavior of the form $\langle V_A\rangle/\langle V\rangle \sim R^{\alpha}$, with $\alpha \sim -1$ is observed up to $R \sim 0.4$ AU, while $\langle V_A\rangle/\langle V\rangle \sim 0.15 \pm 0.03$ at larger distances. The location of this breakdown of the scaling features is consistent with previous observations showing a dynamical phase transition in the power spectral exponents (9), in the fractal topology of the magnetic field (12), and in entropic-based measures (28). The above findings have been interpreted as a continuous transition between two different

Significance Statement

The evolution of solar wind turbulence in the inner Heliosphere is one of the long-standing fundamental problems of space plasma physics. We propose how to reconcile recent observations by the Parker Solar Probe with expectations from the magnetohydrodynamic theory. Our study reveals a magnetically–dominated scenario in the innermost Heliosphere and a fluid–like at larger distances due to the concurrent effects between stochastic magnetic field fluctuations and plasma thermal expansion affecting the distribution between magnetic and kinetic energy. This is a promising approach for interpreting the results of in-situ spacecraft measurements.

.

T.A. designed the study, developed the theoretical framework and provided a first draft of the manuscript. S.B. developed the numerical algorithms and performed data analysis. G.C. contributed to the interpretation of the results and revised the manuscript. M.S. prepared the data set and contributed to the development of numerical algorithms. R.B. contributed to the theoretical framework and the interpretation of the results.

The authors declare no competing interests.

---

[1] To whom correspondence should be addressed. E-mail: tommaso.albertiinaf.it

states, mainly driven by the role of the magnetic field intensity or plasma β that can inhibit some degrees of freedom of the system, thus producing a topological change from a forced turbulence to decaying turbulence (9, 12). To exploit the nature of these two states of turbulence we evaluate the joint distribution of the values of the cross-helicity σ_C and the residual energy σ_R at different heliocentric distances (see Fig. 3).

Moving away from the Sun σ_C evolves from values larger than 0.5 to values close to 0; conversely, σ_R moves from negative to positive values. The observed decreases in the cross-helicity σ_C suggests an evolution from a more to a less Alfvénic turbulence, while the observed trend for the residual energy σ_R suggests a switch from a more magnetically–dominated state toward a kinetically–dominated one. In details, at heliocentric distances less than 0.4 AU the observed scenario lies in the plane $(\sigma_C > 0.5, \sigma_R < 0)$. This suggests the predominance of outward Alfvénic fluctuations with a magnetically–dominated scenario. Going away from the Sun, this scenario still persists $(\sigma_R < 0)$ but with an increase in the occurrence of inward fluctuations $(\sigma_C < 0)$. Between 0.4 AU and 0.6 AU a transition region seems to be reached when no clear/dominating σ_C–σ_R patterns are observed. What emerges is a progressive reduction in the occurrence of the state $(\sigma_C > 0.5, \sigma_R < 0)$, leaving the floor to a $(\sigma_C \gtrsim 0, \sigma_R > 0)$ state. This corresponds to a more kinetically–dominated energy budget with a non-Alfvénic nature of fluctuations (or a reduced frequency of occurrence of Alfvénic states). Finally, at larger distances (i.e., $R > 0.6$ AU) the most probable state is $(\sigma_C, \sigma_R) = (0, 1)$, corresponding to a kinetically–driven scenario. Thus, our findings are in agreement with the simple theoretical framework and describe a transition between two states that can be then classified as:

(i) $\sigma_C^2 + \sigma_R^2 = 1$ with $\sigma_C > 0$ & $\sigma_R < 0$ for $R < 0.4$ AU: this corresponds to a magnetically–dominated state with the predominance of outward Alfvénic fluctuations;

(ii) $\sigma_R = 1$ for $R > 0.6$ AU: this corresponds to the predominance of a kinetically–dominated state with non-Alfvénic fluctuations.

Conclusions

As a final task we discuss implications of our findings, trying to interpret them in the framework of turbulence. Earlier studies (e.g., 9, 12) using PSP observations have shown that an MHD Alfvénic scenario is reached when approaching the Sun for the spectral and the scaling properties of the Elsässer field fluctuations, although mainly dominated by one Alfvénic mode (specifically, z^+), as well as for both the magnetic and the velocity field fluctuations across the inertial range, with a spectral exponent close to –3/2 (9). Conversely, at distances larger than 0.4-0.5 AU all fields are characterized by a spectral exponent close to –5/3 (9), and both Alfvénic modes are almost equi-probable $(|\mathbf{z}^+|/|\mathbf{z}^-| \sim 1)$. According to the earlier work by (25) an initially asymmetric MHD turbulence $|\mathbf{z}^+| \gg |\mathbf{z}^-|$, like that observed close to the Sun by PSP (9), in absence of nonlinear interactions, should relax toward a state characterized by the presence of only one of the possible modes $\mathbf{z}^+$ or $\mathbf{z}^-$. Our observations based on the σ_C–σ_R distribution seem to suggest a slightly modified framework. Indeed, an initially asymmetric state $(\sigma_C > 0)$ relaxes toward a state with non-Alfvénic fluctuations $(\sigma_C = 0)$. This is in agreement with our scenarios describing a transition from a state $(\sigma_C, \sigma_R) = (> 0, \lesssim 0)$ toward a state $(\sigma_C, \sigma_R) = (\sim 0, \to 1)$. Furthermore, according to (25) both the initial and the final state should be characterized by the same spectral exponent, exactly matching that of an Iroshninov-Kraichnan picture of turbulence $\beta = -3/2$ (29, 30). As shown by (9) this does not occur, being $\beta = -3/2$ close to the Sun (< 0.4 AU) and $\beta = -5/3$ far-away from the Sun (>0.6 AU). Thus, PSP observations seem to suggest that the relaxation is from an initially asymmetric turbulence *à la* Iroshninov-Kraichnan toward a symmetric state *à la* Kolmogorov. We indeed demonstrated that the final state is not characterized by the absence of one of the two Alfvénic modes but that we are observing a different nature of the $\mathbf{v}$–$\mathbf{b}$ coupling, linked to the more/less Alfvénic nature of the solar wind close/far-away from the Sun (9). This explains why close to the Sun an MHD Alfvénic turbulence *à la* Iroshninov-Kraichnan is observed, with a spectral exponent –3/2, while close to the Earth a kinetic (fluid) turbulence scenario *à la* Kolmogorov, with $\beta = -5/3$, can be drawn. Our results have fundamental implications in the field of turbulence, both for modelling approaches and for observational results. A new framework for interpreting the role of intermittency, markedly observed close to the Earth, in terms of fluid–like scenarios is needed. Conversely, the global self-similar nature of the field fluctuations across the inertial range close to the Sun needs to be described in a magnetically-dominated scenario. More efforts are needed to describe the evolution of the helical component of turbulence in the inner heliosphere that cannot be interpreted in a simple transport–like scenario but needs to be properly framed out in an evolving scenario.

Our results needs to be further assessed with more and more PSP orbits as well as with observations of the sub-Alfvénic region that could open a completely different framework for the early stages of the solar wind turbulence evolution when leaving the Sun. A critical view of the role of the turbulent cascade in the solar wind is needed, searching for novel models of the solar wind expansion that could be at the basis of the observed scenarios. Indeed, it has been recently demonstrated how including the expansion in solar wind modeling allows to observe nearly equal spectral exponents for the Elsässer fields, as observed, also reproducing the observed variability of spectral indices at larger distances (15).

ACKNOWLEDGMENTS. We acknowledge the NASA Parker Solar Probe Mission and the SWEAP team led by J. Kasper and the FIELDS team led by S. D. Bale for use of data. The data can be downloaded from the NASA CDAWeb (https://cdaweb.gsfc.nasa.gov/pub/data/psp/). This work is funded by the Italian MIUR-PRIN grant 2017APKP7T on "Circumterrestrial Environment: Impact of Sun-Earth Interaction". M.S. acknowledges the PhD course in Astronomy, Astrophysics and Space Science of the University of Rome "Sapienza", University of Rome "Tor Vergata" and Italian National Institute for Astrophysics (INAF), Italy.

1. N Fox, et al., The solar probe plus mission: Humanity's first visit to our star. *Space Sci. Rev.* **204**, 7–48 (2016).

2. JC Kasper, et al., Alfvénic velocity spikes and rotational flows in the near-Sun solar wind. *Nature* **576**, 228–231 (2019).
3. SD Bale, et al., Highly structured slow solar wind emerging from an equatorial coronal hole. *Nature* **576**, 237–242 (2019).
4. DM Malaspina, et al., Plasma Waves near the Electron Cyclotron Frequency in the Near-Sun Solar Wind. *The Astrophys. J. Suppl. Ser.* **246**, 21 (2020).
5. R Chhiber, et al., Clustering of intermittent magnetic and flow structures near parker solar probe's first perihelion—a partial-variance-of-increments analysis. *The Astrophys. J. Suppl. Ser.* **246**, 31 (2020).
6. TD de Wit, et al., Switchbacks in the near-sun magnetic field: long memory and impact on the turbulence cascade. *The Astrophys. J. Suppl. Ser.* **246**, 39 (2020).
7. A Lotekar, et al., Kinetic-scale current sheets in near-Sun solar wind: properties, scale-dependent features and reconnection onset. *arXiv e-prints* p. arXiv:2202.12341 (2022).
8. R Chhiber, WH Matthaeus, TA Bowen, SD Bale, Subproton-scale intermittency in near-sun solar wind turbulence observed by the parker solar probe. *The Astrophys. J. Lett.* **911**, L7 (2021).
9. C Chen, et al., The evolution and role of solar wind turbulence in the inner heliosphere. *The Astrophys. J. Suppl. Ser.* **246**, 53 (2020).
10. RC Allen, et al., Solar wind streams and stream interaction regions observed by the parker solar probe with corresponding observations at 1 au. *The Astrophys. J. Suppl. Ser.* **246**, 36 (2020).
11. ME Cuesta, TN Parashar, R Chhiber, WH Matthaeus, Intermittency in the Expanding Solar Wind: Observations from Parker Solar Probe (0.16au), Helios 1 (0.3-1au), and Voyager 1 (1-10au). *arXiv e-prints* p. arXiv:2202.01874 (2022).
12. T Alberti, et al., On the scaling properties of magnetic-field fluctuations through the inner heliosphere. *The Astrophys. J.* **902**, 84 (2020).
13. T Alberti, et al., The "Singular" Behavior of the Solar Wind Scaling Features during Parker Solar Probe-BepiColombo Radial Alignment. *The Astrophys. J.* **926**, 174 (2022).
14. A Verdini, et al., Numerical simulations of high cross-helicity turbulence from 0.2 to 1 AU. *Nuovo Cimento C Geophys. Space Phys. C* **42**, 17 (2019).
15. R Grappin, A Verdini, WC Müller, Spectral evolution of Alfvénic turbulence in *SF2A-2021: Proceedings of the Annual meeting of the French Society of Astronomy and Astrophysics. Eds.: A. Siebert*, eds. A Siebert, et al. pp. 222–225 (2021).
16. P Hellinger, et al., Plasma Turbulence and Kinetic Instabilities at Ion Scales in the Expanding Solar Wind. *The Astrophys. J. Lett.* **811**, L32 (2015).
17. U Frisch, From global scaling, a la Kolmogorov, to local multifractal scaling in fully developed turbulence. *Proc. Royal Soc. Lond. Ser. A* **434**, 89–99 (1991).
18. V Carbone, Cascade model for intermittency in fully developed magnetohydrodynamic turbulence. *Phys. Rev. Lett.* **71**, 1546–1548 (1993).
19. V Carbone, Scalings, Cascade and Intermittency in Solar Wind Turbulence. *Space Sci. Rev.* **172**, 343–360 (2012).
20. G Parisi, U Frisch, *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics*, eds. M Ghil, R Benzi, G Parisi. (North-Holland Publ. Co., Amsterdam/New York), p. 449 (1985).
21. R Benzi, G Paladin, A Vulpiani, G Parisi, On the multifractal nature of fully developed turbulence and chaotic systems. *J. Phys. A: Math. Gen.* **17**, 3521–3531 (1984).
22. R Benzi, et al., Extended self-similarity in turbulent flows. *Phys. Rev. E* **48**, R29 (1993).
23. G Boffetta, A Mazzino, A Vulpiani, TOPICAL REVIEW: Twenty-five years of multifractals in fully developed turbulence: a tribute to Giovanni Paladin. *J. Phys. A: Math. Gen.* **41**, 363001 (2008).
24. WM Elsasser, The Hydromagnetic Equations. *Phys. Rev.* **79**, 183–183 (1950).
25. M Dobrowolny, A Mangeney, P Veltri, Fully Developed Anisotropic Hydromagnetic Turbulence in Interplanetary Space. *Phys. Rev. Lett.* **45**, 144–147 (1980).
26. S Bale, et al., The fields instrument suite for solar probe plus. *Space Sci. Rev.* **204**, 49–82 (2016).
27. EN Parker, Dynamics of the Interplanetary Gas and Magnetic Fields. *The Astrophys. J.* **128**, 664 (1958).
28. M Stumpo, V Quattrociocchi, S Benella, T Alberti, G Consolini, Self-Organization through the Inner Heliosphere: Insights from Parker Solar Probe. *Atmosphere* **12**, 321 (2021).
29. PS Iroshnikov, Turbulence of a Conducting Fluid in a Strong Magnetic Field. *Sov. Astron.* **7**, 566 (1964).
30. RH Kraichnan, Inertial-Range Spectrum of Hydromagnetic Turbulence. *Phys. Fluids* **8**, 1385–1387 (1965).

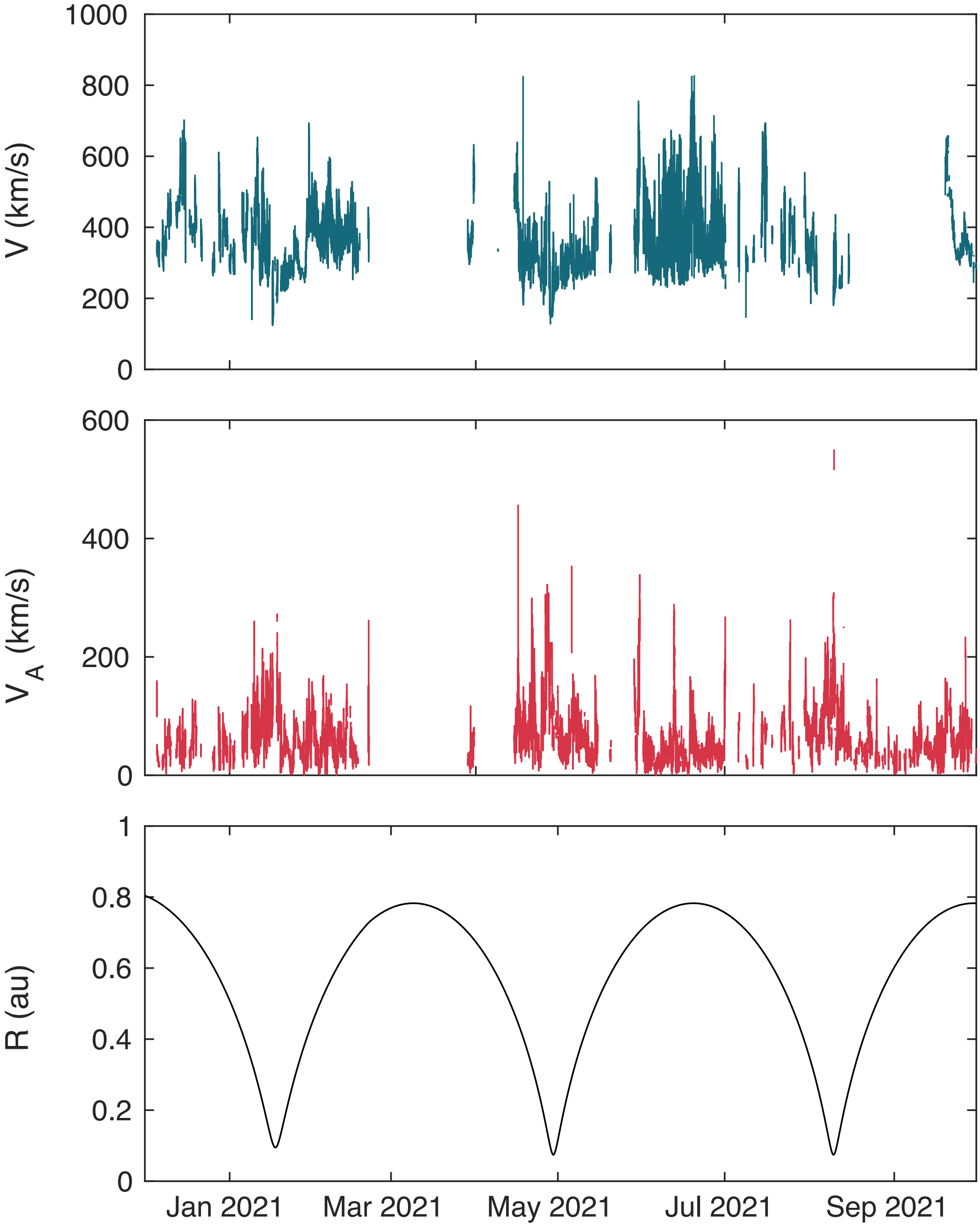

Fig. 1. (From top to bottom) The plasma bulk speed V, the Alfvén speed V_A, and the PSP radial distance to the Sun R.

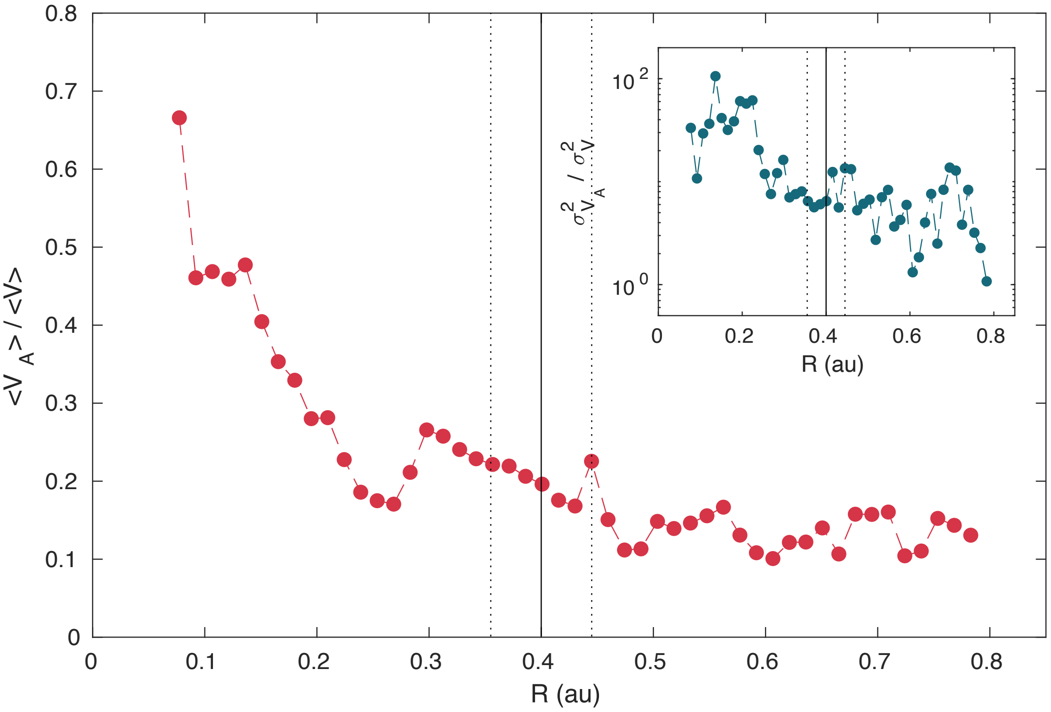

Fig. 2. The radial dependence of the ratio between the Alfvén speed V_A and the plasma bulk speed V. The vertical solid and dashed lines refer to the transition region $R = [0.420 \pm 0.045]$ AU as in (28). The inset shows the ratio between their variances.

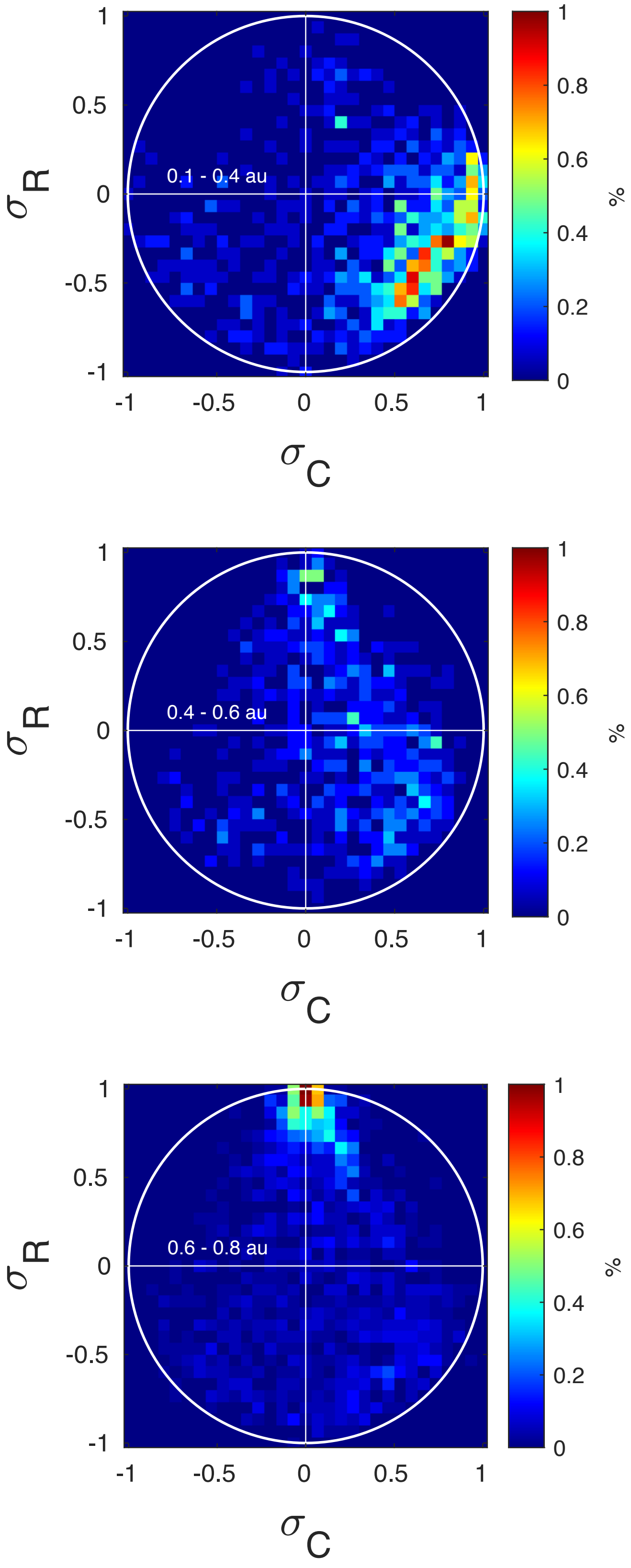

Fig. 3. The joint distribution of the values of the normalized cross-helicity σ_C and the normalized residual energy σ_R at different heliocentric distances.